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J.R. Hicks on Ordinal Approach to Consumer Equilibrium: Some Critical Observations

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Abstract: J.R. Hicks is the author of the ordinal approach to consumer equilibrium on which the modern demand theory is founded. Responding to the general criticism against the cardinal approach developed by two fellow predecessors – Stanley Jevons and Alfred Marshall – he claims that consumer equilibrium can be obtained without quantifying utility. This condition is the equality between MRT and MRS. Measuring MRS is not necessary because the value of MRT is known. We argue that the theoretical and empirical merits of Hicks' claim are contestable because the MRS is the ratio of two cardinal concepts, the marginal utilities of X and Y.

Keywords: Consumer Theory; Cardinal vs. Ordinal Approach; Alfred Marshall, Pareto and J.R. Hicks.

Highlights

- Hicks is the acclaimed author of the ordinal approach used in modern microeconomics.
- This paper argues that Hicks' theory demands critical review for two reasons:
  - First, the slope of the indifference curve is the ratio of two marginal utilities, which are cardinal concepts.
  - Second, the slope of the budget line is the ratio of two commodity prices, meaning they are cardinal too.
- Therefore, the equilibrium point indicated by the tangency between the budget line and the indifference curve is cardinal, not ordinal.

1. Introduction

The microeconomic theory of consumer behaviour explains how the consumer, referred to as *Homo economicus* (HE), chooses a combination of commodities from a market basket that lists all the goods and services they need and desire. HE is forced to choose because their time-bound budget does not allow them to satisfy all these needs and desires. Given that they are a utility maximiser, they choose the combination that maximises their total utility. This is done through equalising the marginal utilities of all goods and services included in the market basket. In economic terminology, the combination represents HE's equilibrium commodity bundle. Once this equilibrium bundle is found, it is

used to derive an individual demand curve, which is the ultimate objective of consumer theory.

Currently, we have two approaches - cardinal and ordinal – that demonstrate how the law of individual demand (LoID) can be constructed. This paper concerns the ordinal approach to deriving a demand curve for HE. This curve is acronymed as D_{HE}. In other words, we will use D_{HE} instead of LoID throughout the paper.

The cardinal version deals with a single commodity and derives D_{HE} directly. The consumer equilibrium is given by the equimarginal principle, which states that the ratios of marginal utilities to prices of all commodities must equal the marginal utility of income spent on each commodity. Hicks (1939) argues that the equimarginal principle is unsound for two reasons. First, holding the marginal utility of income constant is unreasonable when the prices of several commodities change simultaneously or when the prices of necessary commodities change abruptly. Second, the interpersonal comparison of the marginal utility of income is not scientific.

In his book, *Wealth and Welfare*, Hicks (1981) details the history and the analytical background of the ordinal approach. His contribution began with his article 'A Reconsideration of the Theory of Value' published in the *Economica* (Hicks & Allen, 1934), which was inspired by Lerner's (1933) paper 'The Diagrammatical Representation of Elasticity of Demand' published in *The Review of Economic Studies*. In that paper, Lerner argues that the curvature of an isoquant can be interpreted as the elasticity of substitution between two inputs. Since an indifference curve shares all the properties of an isoquant, Hicks argued that its curvature should be interpreted in the same way. More specifically, he derived a formula for price elasticity of demand (η) that connects both income elasticity (ξ) and the price elasticity of substitution between two goods:

$$\eta \text{ of } X = k\,\xi + (1-k)\,(\text{e.s.}),$$ where k is the proportion of the total budget spent on X.

With this idea in mind, Hicks replaces Marshall's cardinal utility approach with his proposal to avoid subjective valuation of consumer utility. Marshall argued that subjective utility can be indicated by hypothetical commodity prices using diminishing marginal utility theory. However, this hypothetical price schedule becomes unnecessary for deriving individual demand curves when real prices and income are used from the budget line. More specifically, Hicks used the marginal rate of transformation (MRT) to represent MRS. In Hicks' words:

> *By transforming the subjective theory of value into a general logic of choice, they [Fisher (1892) and Pareto (1906)] extend its applicability over wide fields of human conduct.*

> *Two opportunities for the exercise of this new freedom seem of particular importance for the future of economics. One is the Economic Theory of the State, where the shackles of utilitarianism have always galled; the other is the Theory of Risk, where the application of the same logic seems fundamental to any progress in economic dynamics (Hicks & Allen, 1934, p. 54).*

Second, Hicks examines the adjustments made necessary by Pareto's discovery in the statement of the marginal theory of value.

> *Much of his theory had already been constructed before he realised the immeasurability of utility, and he never really undertook the labour of reconstruction that his discovery had made necessary. What has now to be done is to take in turn a number of the main concepts which the subjective theory has evolved; to examine which of them are affected by the immeasurability of utility; and of those which have to be abandoned, to enquire what, if anything, can be put in their place. It is hoped in this way to assist in the construction of a theory of value in which all concepts that pretend to be quantitative exactitude can be rigidly and precisely defined (Hicks & Allen, 1934, pp. 54-55).*

In Marshall's demand system, the individual demand curve slopes downward because of the LDMU, which stems directly from assumptions about the nature of TU and MU. In this theory, TU increases at a decreasing rate as the stock of the concerned commodity increases. This assumption about TU, in turn, leads MU to decrease to zero or even negative.

In Hicks' system, the slope of the indifference curve (IC) is given by the ratio of two marginal utilities, with the IC convex to the origin and the individual demand curve taking its expected shape. Thus, the convexity of the IC is necessary for the demand curve to slope downward, which we can obtain by retaining the validity of Marshall's LDMU.

This mathematical condition raises two analytical issues. First, how does Hicks make his approach ordinal by using cardinal arguments? Second, Marshall's theory of demand applies only to commodities related to physical pain and pleasure. This fact naturally raises questions about Hick's reasons and justifications concerning the slope of the IC.

To find answers to these questions, we will review Hicks' two publications – Hicks and Allen (1934) and Hicks (1939). The following section summarises Hicks' interpretation of Marshall's demand theory. Section 3 does the same for Pareto, as Hicks borrows Pareto's theory of indifference curves to develop his new approach to consumer theory. Section 4 describes how Hicks develops his theory of consumer equilibrium, which eventually leads to the construction of D_{HE}. The theory is criticised in Section 5, and the paper is concluded in Section 6.

2. Hicks on Marshall

Hicks begins developing his ordinal approach by criticising Marshall's utility concepts, which constitute the building blocks of his demand system. The first term that Hicks censures is the MU_X. It is not meaningful because TU_X is not quantitatively definable.

> *But the theory of value does not need any precise definition of marginal utility. What it does need is only this: when an individual's system of wants is given, and he possesses any given set of goods, X, Y, Z..., we should know his marginal rate of substitution between any two goods. The marginal rate of substitution of any good Y for any other good X is defined as the quantity of good Y which would just compensate him for the loss of a marginal unit of X (Hicks & Allen, 1934, p. 55).*

This terminology, MRS, is not new; it is the ratio of the marginal utilities of X and Y, which may be called the 'relative marginal utility.' Hicks replaced this terminology to avoid confusion between the Marshallian conceptions of utility and his own. Second, he claims some advantages in concentrating on the substitutional character of the concept at this early stage of development. The MRS between any two goods must equal the ratio of their prices if the consumer is to be in equilibrium concerning a system of market prices.

> *When quantities of X and Y are represented on an indifference diagram (quantities of all other goods possessed being therefore supposed given), the marginal rate of substitution between X and Y is measured by the slope of the indifference curve which passes through the point at which the individual is situated. This depends simply upon the system of indifference curves; given the indifference map, we can read off directly the slope at any point; given the slopes at all points within a region, we can reconstruct the indifference map for that region (Hicks & Allen, 1934, p. 56).*

Next, Hicks dismisses Marshallian terminology, LDMU. Since the DMRS has substituted the DMU, it is logical that the LDMRS should replace the LDMU. This, however, is not the interesting point in Hick's replacement procedure; the interesting point is Hick's claim that this replacement is something more than merely changing the terminology.

> *When we seek to translate the principle of diminishing marginal utility into definable terms, it does not appear at first sight evident that this is the condition we must use. Moreover, it is an interesting historical fact that when Pareto found himself confronted with this question, he, first of all, gave the condition that the indifference curves must be convex to the origin, and then went on to add a further condition: that the marginal rate of substitution will increase, not only when Y is substituted for X, but also when the supply of Y is increased without any reduction in the supply of X. This condition looks as good a translation of diminishing marginal utility as the other, but (as Pareto realized) it*

> *stands on an altogether different footing. Cases that do not satisfy this latter principle undoubtedly exist in plenty, and there is no particular difficulty in fitting them into a general theory. Exceptions to the true principle of increasing the marginal rate of substitution would be much more serious.*
>
> *For it is certain that for a position to be one of stable equilibrium at given prices, the marginal rate of substitution at that point must be increasing. If it is not, then, even if the marginal rate of substitution equals the price ratio, it does not provide any appreciable advantage to the marginal utility of one commodity over another. Equilibrium would be unstable - the individual would be at a point of minimum, not maximum, satisfaction.*
>
> *The assumption that the principle of increasing the marginal rate of substitution is universally true means simply that any point, throughout the region we are considering, might be a point of equilibrium with appropriate prices.*
>
> *There must be some points at which it is true, or we could get no equilibrium at all. To assume it true universally is a serious assumption, but one which seems justifiable until significant facts are adduced which make it necessary for us to pay careful attention to exceptions' (Hicks & Allen, 1934, p. 56).*

This is all that Hicks said in his 1934 article, co-authored by R.G.D. Allen. However, he summarised Marshall's model in the 1939 book, which is interesting for two reasons. First, it allows us to see Hicks's interpretation of Marshall's economics in general and the demand theory in particular. Second, it also shows his motive for economic analysis for the same purposes. More specifically, it shows how he uses the concepts and rules of mathematics, a formal science, to manipulate those of economics, a social science. His interpretation of Marshall's model is quoted below:

> *A consumer with a given money income is confronted with a market for consumption goods, on which the prices of those goods are already determined; the question is, how will he divide his expenditure among the different goods? For convenience, it is assumed that the goods are available in minimal units. It is assumed that the consumer derives from the goods she purchases so much 'utility', the amount of utility being a function of the quantities of goods acquired, and that she will spend her income in such a way as to bring in the maximum possible amount of utility. However, utility will be maximised when the marginal unit of expenditure in each direction brings in the same increment of utility. For if this is not so, a transference of expenditure from one direction to another will involve a greater loss of utility in the direction where expenditure is reduced than will be compensated by the gain in utility in the direction where expenditure is increased (from the principle of diminishing marginal utility). The total utility must therefore be diminished, whatever transfer is made. Since, with small units, the differences between the marginal utilities of two successive units of a commodity may be neglected, we can express the conclusion in another way: the marginal utilities of the various commodities bought must be proportional to their prices.*

> *Marshall's argument, therefore, proceeds from the notion of maximising total utility, by way of the law of diminishing marginal utility, to the conclusion that the marginal utilities of commodities bought must be proportional to their prices (Hicks, 1939, pp. 11-12).*

The difficulty with Marshall's theory, Hicks says, is that he did not explain the meaning of 'utility' that the consumer maximises. Nor did he explain the exact basis for the law of diminishing marginal utility.

3. Hicks on Pareto

Marshall's cardinal approach involves analysing a single commodity that eventually leads to the derivation of its demand curve. This technique, therefore, cannot handle interrelations among multiple commodities across different markets. The problem of value theory in economics, Hicks argues, is to analyse interrelations among related commodity markets. Pareto (1906) did this by focusing his attention on the problems of complementary and competitive goods. For this purpose, Pareto borrowed Edgeworth's (1881) geometrical device, the indifference curve, which is appropriate for analysing the ranking of the combinations between two commodities. When there are two commodities, X and Y, a consumer can have three choices: X_0 P Y_0, Y_0 P X_0, or X_0 I Y_0. The IC technique uses the last-choice condition, which states that the consumer is indifferent between the quantities X_0 and Y_0 because the two commodities yield the same level of utility. Given this level of utility, it is mathematically possible to make other combinations, which will also satisfy this criterion. Here, the level of utility is fixed, while the quantities of two commodities are variable.

The fundamental question now is the shape of the IC. From the mathematical point of view, the nature of IC's shape is evident. It must be convex to the origin because the amount of the commodity on the vertical axis must decrease when that on the horizontal axis increases. In the words of Hicks:

> *The slope of the curve passing through any point P [Fig. 2, p. 15] has indeed a very definite and important meaning. It is the amount of Y which is needed by the individual to compensate him for the loss of a small unit of X. Now the gain in utility obtained by gaining such an amount of Y equals the amount of Y gained × marginal utility of Y; the loss in utility got from losing the corresponding amount of X equals the amount of X lost × marginal utility of X (so long as the quantities are small). Therefore, since the gain equals the loss, the slope of the curve is.*
>
> *amount of Y gained ÷ amount of X lost = marginal utility of X ÷ marginal utility of Y*
>
> *The slope of the curve passing through P measures the ratio of the marginal utility of X to the marginal utility of Y when the individual has quantities PM and PN of X and Y, respectively (Hicks, 1939, p. 14).*

Since the ratio of two marginal utilities determines the slope of an IC, this raises a question about the relationship between the MU and IC curves. More specifically, can we translate the principle of diminishing marginal utility in terms of this diagram? The principles of diminishing marginal utility and convexity of an indifference curve, Hicks argues, are not the same thing. They appear the same because of the direct effects that are considered in the cardinal approach. When X increases, its MU decreases, and when Y decreases, its MU increases. Therefore, IC's slope must diminish, meaning IC must be convex to the origin. However, in the two-commodity case, there could be significant indirect effects. The increase in X may affect not only the marginal utility of X but also the marginal utility of Y. With such related goods, the above argument does not necessarily follow. Suppose that the increase in X lowers the marginal utility of Y, and the diminution in Y raises the marginal utility of X and that these cross-effects are considerable. Then the cross-effects may offset the direct effects, and a movement along the indifference curve to the right may increase the slope of the curve. This is no doubt a very queer case, but it is consistent with diminishing marginal utility. Diminishing marginal utility and convexity of the indifference curves are not the same thing.

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Hicks' ultimate objective in the theory of consumer behaviour is to determine the conditions of consumer equilibrium, which requires him to introduce two additional sets of information. First, the consumer, already assumed, has more wants than she can satisfy with her limited income. This assumption requires introducing a budget constraint in the model. Second, we need the market prices of X and Y, which our consumer will take as given. Using these two sets of information, we can construct a budget line that shows the attainable level of IC on the consumer's indifference map. Any indifference curve that does not cut this budget line is unavailable, meaning any that do are available. Among those ICs cutting the budget line, the tangent one maximises utility. This tangency indicates that the consumer attains equilibrium at the point where the slopes of the IC and the budget line are equal, i.e., the MRS between X and Y equals the price ratio of X and Y.

This tangency condition, Hicks argues, indicates the possibility of determining an equilibrium level of consumption without any information about the utility the consumer receives from buying different bundles of two goods. Two points may be noted here. First, Marshall's model requires us to know the consumer's utility surface (TU). In contrast, Pareto's theory only assumes we need to know her indifference map, which contains less information than the utility surface. Second, Marshall's theory requires maximising a given utility function that involves a given intensity of desire for any collection of goods. This assumption, Hicks says, has made many people uncomfortable. Pareto's model does not require this assumption because the indifference map shows the scale of preferences.

4. Hicks' Hypothesis of Ordinal Utility Theory

Hicks' contribution begins from this point. Describing Pareto's discovery as opening a new window into demand theory, he argues that there are strong technical and economic reasons to take advantage of this opportunity.

> *The quantitative concept of utility is not necessary to explain market phenomena. Therefore, on the principle of Occam's razor, it is better to do without it. For it is not, in practice, a matter of indifference if a theory contains unnecessary entities. Such entities are irrelevant to the problem at hand, and their presence is likely to obscure the vision. How important this is can only be shown by experience; I shall hope to convince the reader that it is of some considerable importance in this case (Hicks, 1946, p. 18).*

To pursue his line of reasoning, Hicks leaves Pareto aside, arguing that his analysis does not help the present research, which aims to explore the possibility of developing a complete theory of consumer demand, avoiding Marshall's notion of quantitative utility.

> *In constructing such a theory, it will be necessary every time to reject any concept which is at all dependent upon quantitative utility so that it cannot be derived from the indifference map alone. We start from the indifference map alone; nothing more can be allowed (Hicks, 1946, p. 18).*

As described above, Hicks rejects Marshall's two fundamental concepts - MU and LDMU – and replaces them with MRS and LDMRS. However, he needs to justify the LDMRS's rationale as a necessary condition for attaining a stable consumer equilibrium.

> *The justification is this. We need the principle of diminishing marginal rate of substitution for the same reason that Marshall's theory needed the principle of diminishing marginal utility. Unless, at the point of equilibrium, the marginal rate of substitution is diminishing, the equilibrium will not be stable. Even if the marginal rate of substitution equals the*

> *price ratio, so that the acquisition of one unit of X would not yield any appreciable advantage, nevertheless, if the marginal rate of substitution is increasing, the acquisition of a larger quantity would be advantageous (Hicks, 1946, p. 21).*

This justification seems more mathematical than economic, suggesting that Hicks needs the principle to have general validity. The LDMU has general validity, as important economic conclusions have been drawn from it. Accordingly, Hicks reexamines those conclusions to see whether they apply to the IC approach.

> *What were, in fact, the grounds upon which economists used to base their general principle of diminishing marginal utility? Usually an appeal to experience, though to the experience of that uncomfortably vague sort that does not offer any opportunity for actual testing. Critics have not been lacking in pointing out that this procedure was not very scientific, and the doubts which have been thrown by our present discussion upon the intelligibility of the law of diminishing marginal utility itself can only strengthen the case against the traditional procedure. If, however, we throw over diminishing marginal utility as being, in any case, dubious, and now certainly irrelevant, can we base upon similar experience a general principle of diminishing marginal rate of substitution? Again, I suppose, we might get away without being challenged, but one would like a surer foundation (Hicks, 1946, p. 22).*

Hicks' explanation of this 'surer foundation' is critically important to understanding his ordinal approach. Accordingly, the entire section is quoted below.

> *We can, I think, get that surer foundation if we reflect on the purpose for which we require our principle. We want to deduce from its laws of market conduct — laws, that is, which deal with the reaction of the consumer to changes in market conditions. When market conditions change, the consumer moves from one point of equilibrium to another point of equilibrium; at each of these positions, the condition of diminishing marginal rate of substitution must hold, or he could not take up such a position at all. So much is clear directly, but to proceed from this to the law of diminishing marginal rate of substitution, as we need it in economic theory, an assumption is necessary. We must assume that the condition holds at all intermediate points, so that there are no kinks in the curves between the two equilibrium positions. (If there are kinks in the curves, curious consequences follow, such that there will be some systems of prices at which the consumer will be unable to choose between two different ways of spending his income.) The general principle of diminishing the marginal rate of substitution merely rules out these oddities; by that principle, we select the simplest of the various possibilities before us.*
>
> *As we go on, we shall find that most of the 'laws' of pure economic theory can be looked at in this sort of way. Pure economics has a remarkable way of producing rabbits out of a hat—an a priori proposition that refers to reality. It is fascinating to try to discover how the rabbits got in; for those of us who do not believe in magic, we must be convinced that they got in somehow. I have become convinced myself that they get in two ways.*

> *One is by the assumption, at the beginning of every economic argument, that the things to be dealt with in the argument are the only things that matter in some practical problem. (This is always a dangerous assumption, and nearly always more or less wrong — which is why the application of economic theory is such a ticklish matter.) That takes us much of the way, but it does not take us the whole way. The other assumption is that which we have just isolated, the assumption that kinks can be neglected, that there is a sufficient degree of regularity in the system of wants (and also, as we shall see later, in the productive system) for any set of quantities in the neighbourhood of those with which we are concerned to be a possible position of equilibrium at some system of prices. Again, this assumption may be wrong, but being the simplest possible, it is a reasonable starting point, and its accord with experience seems definitely reasonable.*
>
> *The road which lies before us now begins to be distinguishable. If this is the proper foundation of the principle of diminishing the marginal rate of substitution among commodities, other principles can be discovered whose foundation is precisely similar. These principles can be enumerated, and their consequences worked out. Some of them deal with production…; the rest are extensions, into one field or another, ... That there are a great many such extensions appears at once when we consider how wide the variety of human choices can be fitted into the framework of the Paretian scale of preference. What begins as an analysis of the consumer's choice among consumption goods ends as a general theory of economic choice. We are in sight of a unifying principle for the whole of economics (Hicks, 1946, pp. 23-24).*

Hicks explains how this two-commodity model can be extended to an n-commodity model with the following sentences:

> *When expenditure is distributed between more than two goods, the indifference diagram loses its simplicity; for three goods, we need three dimensions, and for more than three goods, geometry fails us altogether. However, the principles which we have established… remain substantially unaffected. The marginal rate of substitution can be defined as before, with the added proviso that the quantities consumed of all other commodities (Z) must remain unchanged. The consumer is only in complete equilibrium if the marginal rate of substitution between any two goods equals their price ratio. Over the principle of diminishing marginal rate of substitution, there is a slight difference.*
>
> *So that the equilibrium should be stable, when expenditure is distributed among many commodities, no possible substitution of equal market values must lead the consumer to a preferred position. This means not only that we must have a diminishing marginal rate of substitution between each pair of commodities, but also that more complicated substitutions (of some X for some Y and some Z) must be ruled out in the same way [emphasis added]. We may express this by saying that the marginal rate of substitution must diminish for substitutions in every direction. This is a rather complicated condition, but it will appear, as we proceed, that it leads directly to conclusions of great importance.*
>
> *On the same grounds as before, we shall assume that the marginal rate of substitution diminishes in every direction at every position with which we shall be concerned in our*

> *analysis. I do not think this could be established introspectively, or from 'experience but it can be justified in the same way as we have justified the simpler condition. It becomes clear now, however, that it is a fairly drastic hypothesis, which gives us a good deal to go on, and from which we can expect to deduce some positive results (Hicks, 1946, pp. 24-25).*

5. Some Critical Comments

The primary purpose of this paper is to examine Hicks' claim that a full-fledged theory of demand can be articulated, avoiding the criticisms laid against Marshall's model. The preceding sections have summarised Hicks' arguments in this regard. This section reviews those arguments to make an informed judgment.

However, before getting drawn into the debate, it seems worthwhile to underscore the gravity and difficulty of examining this century-old issue. The utility concept has received due attention in the relevant literature because it lies at the heart of the neoclassical economic model. Consider Samuelson's (1938; 1948) revealed-preference theory, which provides a structured approach to analysing demand behaviour.

Here, Samuelson lays down the principle that a consumer's observed choices reveal her underlying preferences. Suppose a consumer is observed to have chosen a specific consumption bundle X, while another bundle Y was also available. In that case, she reveals her preference for X over Y. Equivalently, we say that X is revealed as preferred over Y. In this manner, choices reveal the consumer's underlying preferences (Demuync & Hjertstrand, 2020).

Rothbard (1956) observed that the utility theory, which analyses the laws of the values and choices of a consumer, had been foundering in stormy seas in that it had been galloping off in several directions at the same time. To salvage and reconstruct the utility theory, he proposed the concept of "demonstrated preference."

> *The concept of demonstrated preference is simply this: that actual choice reveals, or demonstrates a man's preferences; that is, that his preferences are deducible from what he has chosen in action. Thus, if a man chooses to spend an hour at a concert rather than a movie, we deduce that the former was preferred, or ranked higher on his value scale. Similarly, if a man spends five dollars on a shirt, we deduce that he preferred purchasing the shirt to any other uses he could have found for the money. This concept of preference, rooted in real choices, forms the keystone of the logical structure of economic analysis, particularly of utility and welfare analysis (Rothbard, 1956, p. 225).*[1]

A comprehensive analysis of a commodity market in economics began with Marshall; Hicks built on this foundation. However, he had to criticise the

Marshallian approach to promote his version of the consumer theory. Before reviewing those criticisms, let us note some observations on Marshall's demand theory from Elahi and Reardon (2022). Firstly, Marshall's conception of utility seems unclear for three reasons. One, he understands utility as the creation of value, which is a marketing concept, not a consumption one. Two, because of this definition, Hick's statement that utility is not quantitatively measurable becomes confusing. The value created by changing a product's basic features is economic, not psychological. Three, the LDMU primarily applies to commodities directly related to the basic needs of the human body.

Hicks' objections, as noted above, are different. They are concerned about his interpretation of Marshall's model, which inaccurately reflects Marshall's arguments and opinions. More specifically, Hicks has interpreted Marshall considering his equilibrium theory, in which consumers maximise utility subject to the budget constraint. The budget constraint is a necessary condition in Hicks' model; without it, it is impossible to predict consumer equilibrium. Marshall derives the individual demand curve directly from the LDMU theory. An income constraint is not necessary for this model, as it is a hypothetical demand curve that requires only hypothetical commodity prices for its derivation.

Second, Hicks adapts Pareto's IC to illustrate his theory of individual demand D_{HE}, providing the theoretical instruments needed to construct the theory of market demand. Hicks' interpretation is also arguable in this case, as Pareto's objective in building the IC model differed from Hicks'. Pareto spends the first chapter of his book, 'General Principles,' describing different objectives with which political economy is studied. He notes three specific objectives: (i) a collection of recipes applicable to individuals and public authorities in their economic and social activities; (ii) illustrating a doctrine that the author thinks is important for society or humankind; (iii) studying and ascertaining uniformities in nature without expecting any direct benefits to humankind.

> *I should warn the reader that in this Manual, I shall be concerned only with procuring this third objective. This does not imply in the least that I mean to run down or disparage the other two; I only wish to make a distinction among alternative ways of analysing the material and to [indicate the one that will be adopted in this book] (Pareto, 1906, p. 2).*

Pareto addresses the pure theory of exchange in his book, in which all characters are hypothetical. However, Hicks deals with the market demand curve derived from the consumer's equilibrium, which involves the consumer's actual budget and the commodity's price. Accordingly, Hicks's interpretation of Pareto's indifference curve model is subject to criticism. Finally, we will examine Hicks' logic for proposing the LDMRS as the criterion for D_{HE}. Let us first consider

Hicks' objection to MU. He rejects MU, arguing that TU is arbitrary because utility is a psychological phenomenon that cannot be measured scientifically. From a social science perspective, this argument seems sound because economics, as a social science, deals with phenomena involving human action. However, as noted above, Pareto's Manual is not exactly a study in social science, as it deals with issues of pure economics, where empirical details are unimportant. This statement can be supported by repeating Hicks's words: 'the laws of pure economics produce rabbits out of a hat.'

In Hicks' theory, MRS is the rabbit. It is not free of ambiguity and irrationality, which Hicks accuses the Marshallian model of. The MRS is a hypothetical construct, an explanatory variable not directly observable. For example, the concepts of intelligence and motivation are used to explain phenomena in psychology, but neither is directly observable. Cronbach and Meehl (1955) give a more comprehensive definition of a hypothetical construct. It is a concept for which there is not a single observable referent; it cannot be directly observed, and it refers to multiple referents, none of which are all-inclusive. For example, utility is an abstract idea which signifies the satisfaction that the consumer obtains from procuring a commodity. This satisfaction, which economists are obsessed with, refers to the 'want-satisfying' property of the commodity under inquiry.

Accordingly, MRS, the ratio of two marginal utilities, is a hypothetical construct. On the other hand, MRT, being the ratio of two commodity prices, is a market variable. Thus, the tangency between the budget line and an IC indicates the equality between a hypothetical construct and a market variable.

Can we replace a hypothetical variable with an actual one? The answer is 'no.' MRS, i.e., the ratio of two marginal utilities, describes the satisfaction that the consumer receives from buying a specific bundle of two commodities. This buying activity is supposed to be voluntary. However, the bundle indicated by a point on the budget line may not be voluntary; the consumer might be bound by needs to choose the bundle. Accordingly, equating MRS and MRT to avoid the theoretical idea of cardinal utility sounds illogical. If MU and LDMU are unsound concepts, then Hicks's MRS and LDMRS are also unsound.

This principle of logic has eluded our master economists. What then could be the real reason for promoting Hicks' thesis? This apparent logical error originated in the neoclassical scholars' constant efforts, beginning with Jevons, to change the meaning of utility that Bentham attached to the term. Bentham used the word utility to articulate his utilitarian conception of legislation concerning crime and punishment. Punishment, whether physical or mental, is

directly connected with the mind's feeling of 'pain and pleasure.' However, the nature of pain and pleasure that individuals feel from different actions and activities differs significantly. It does not need any explanation to see that the feeling of pain and pleasure from publication, a single action, cannot be compared with that from buying goods and services. Then, the goods and services we use in our daily lives have different capacities to satisfy wants, meaning abstaining from goods and services such as food and medical products and services will cause pain in our lives, whereas forgoing the purchase of diamond ornaments will not.

The final point concerns the root of all these disagreements, which may be traced back to the late 19th century. The first significant author in this regard was John Neville Keynes (1890), who divided the subject matter of political economy into three categories in his classic book, The Scope and Method of Political Economy: positive, normative, and art. In particular, he argued that topics in positive economics are free of subjective judgments, whereas those in normative economics are inevitably judgmental. The analysis in positive economics is theoretical, whereas that in normative economics is policy-related.

Keynes successfully clarified the nature of topics to be included in positive economics, which he called 'what is' issues. However, he did not mention the role that mathematics was supposed to play in this theoretical analysis. Moreover, this was the bone of contention between Marshall and the authors of the marginalist revolution, Stanley Jevons, Maria Edgeworth, Leon Walras, and Vilfredo Pareto.

In this regard, Maffeo Pantaleoni makes interesting observations in his book *Pure Economics* (1898). He argued that the root of all disagreements lies in the failure to maintain the distinction between economics as a science and an art. Economics studied in the 19th century was called pure science because its domain included developing fundamental definitions, theorems, and classifications of economic variables. More specifically, the subject matter of pure economics is entirely theoretical as its methodology is founded on formal logic, in which mathematics plays a significant role. Although Ricardo (1817) is credited with introducing formal or deductive logic in economics, it was Walras (1874) who pioneered the neoclassical adventures in pure economics that eventually overwhelmed the modern economics profession.

In Pantaleoni's view, the subject matter of pure economics and that of political economy are different. His classification, which might seem strange in modern times, is, however, relevant to our discussion because the concept of pure economics entered economics into economic inquiry with Walras. Pareto, who

succeeded Walras, carried on this intellectual project further in his book *Considerations on the Fundamental Principles of Pure Political Economy* (1892 & 1893), which led to the production of the *Manual of Political Economy* (1906). As noted above, Pareto made the objective and nature of his study clear in the first chapter of the *Manual*, which categorically states that the approach introduced in the book is not meant for any practical purpose. Hicks might have overlooked Pareto's message and employed his model as the foundation of the demand theory, which is fundamentally empirical.

Let us look at Hicks' assertions closely. One of the IC assumptions is convexity; without it, the consumer equilibrium will be unstable due to the second-order condition. So, Hicks argues that MRS will diminish, as MU does, due to this theoretical restriction in the model, which creates logical problems in the LDMRS concept. Hicks defends it on technical grounds. Although LDMU has economic justifications, it is rejected as a psychological phenomenon that cannot be measured scientifically. However, he retains this cardinal concept, arguing that we need not quantify it. This argumentation seems pretty much confusing and circular.

Hicks discards LDMU as being dubious and irrelevant, but developed the LDMRS concept based on a similar experiment. He justifies his approach by arguing that the fundamental objective of consumer theory is to deduce laws of market demand that predict how consumers are supposed to behave when market prices change. LDMRS is a mathematical necessity to derive these laws. To give this condition an economic flavour, Hicks imposes additional conditions on the model. The first condition rules out oddities at all intermediate points during the construction of the IC. The second, and most important, assumption in our context concerns the distribution of the consumer's budget across many commodities. He restricts the substitution of equal market values among different commodity groups. This restriction signifies two theoretical conditions that are of enormous practical importance. First, MRS will diminish across each pair of commodities, and second, more complicated combinations, such as some X, some Y, and some Z, are ruled out.

In a mathematical model, these assumptions pose no problems. Therefore, this model seems logically sound when the market demand curve is assumed to be 'notional.' However, Hicks' purpose is to develop a comprehensive demand system as Marshall did. This is an empirical issue that conflicts with theoretical ideas. We immediately accept the proposition that the market demand curve slopes downward because it does not specify any commodity. However, this spontaneous acceptance becomes questionable when we identify the commodity

and the consumer's condition. Consider medical care services. Individuals do not seek medical services unless they become sick. Then, seeking these services depends upon the nature of the sickness. Moreover, buying medical services is conditional on income position. When the cost of medical services decreases, market demand for them might increase, as more patients can access them. However, individual demand is not expected to increase because no one wants to visit health care professionals unless necessary. This is not the situation in which the utility theory that Bentham founded and Jevons applied in political economy.

All objections raised against Hicks seem to have a simple logical explanation: the distinction between the social and formal sciences. Social sciences, including economics and sociology, study human behaviour based on assumed motives suggested by philosophy. This is why all social sciences may be described as 'behavioural sciences.' Here, our job is to observe and collect facts to find uniformity in their relationship. We do not impose any restrictions other than what common sense requires. For example, an average wage earner adjusts her purchase schedules when the prices of the commodities in the food basket change. We are supposed to explain the nature and causes of this change. However, when we impose axioms to certify these changes as rational, the problem no longer remains social; it becomes formal.

On the other hand, formal science deals with invented or imagined ideas which do not exist in the real world (Hume, 1739-40). The primary purpose of formal science is to develop language and theories that other sciences can use to analyse issues they deal with. The reason is that the concepts and theories of formal sciences are true by definition, whereas those of the social sciences are probable.

Mathematics is a handy instrument for economic inquiry, which, however, cannot be its language (Marshall, 1920). This is because, as an empirical science, economics needs to consult facts and experience at every step of generalisation. This is not possible when we use mathematical methods to theorise in economics. The constant, long chain of reasoning might make it challenging to connect the results of the analysis with the problem statement. Then, mathematical models leave little room to compare deductive analysis results with experience, undermining the study's validity.

We lose all the details of consumer theory under budget constraints when constructing a mathematical model that must rely on axioms. The works of Walras, Pareto, Hicks, Samuelson, and Arrow (1951), among the pioneering neoclassical economists, are masterpieces of applied mathematics, but their

relevance and significance in economics are subject to criticism. What is the value of the stability of the perfectly competitive market system in economics when it does not exist? What is the value of Arrow's (1951) social choice theory when his social choice system is incompatible with the existing political system in a democratic country?

6. Conclusion

Marshall developed the widely accepted partial equilibrium model in which the market demand curve slopes downward. This demand curve derives the inverse relationship between the price of a commodity and the quantity demanded from the individual demand curves that constitute it. Furthermore, the individual demand curve gets this property because of the assumption that the degree of utility decreases as the consumer's stock of the commodity increases. This assumption gives rise to two concepts – diminishing marginal utility (DMU) and the law of diminishing marginal utility (LDMU) – which are keystone ideas in Marshall's cardinal approach to consumer theory.

Hicks rejects Marshall's concepts of DMU and LDMU but retains his terminology, MU, to construct the IC technique. The difficulty with Hicks' analysis begins with the ideas of the budget line and the indifference curve. He argues that instead of using Marshall's utility function, he can use Pareto's preference scale to determine the condition of consumer equilibrium. This condition is given by the tangency between the budget line and an indifference curve, where the slopes are the MRT and MRS, respectively. Since MRT is known, Hicks argues that quantifying MRS is unnecessary. However, this argument is logically unsound because MRT is a quantifiable market variable, whereas MRS is a hypothetical construct that is neither observable nor quantifiable. Accordingly, Hicks' derivation of the law of individual demand (D_{HE}) is subject to criticism.

Note

1. Both authors support the Hicksian thesis that utility need not be quantified hypothetically; actual prices and income can serve as indicators of the consumer's subjective utility. This view is contestable. Since this is not a review paper, critical essays are not consulted with the understanding that the points raised in the paper were not discussed in the way they are presented here.

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Appendix

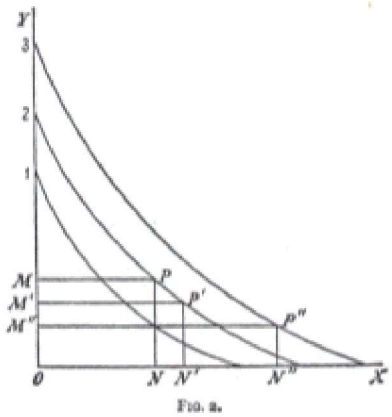

FIG. 2.

Figure: Hicks's Indifference Curve in *Value and Capital*

Source: Hicks, 1939, p. 15